ALGEBRA QUIZ

A

1) $5(3+h)^2 = 5(9+6h+h^2)$
$= \boxed{45+30h+5h^2}$

2) $(x+h)^2 = \boxed{x^2+2xh+h^2}$

3) $6(x+3)-2(x+1)$
$= 6x+18-2x-2$
$= \boxed{4x+16}$

4) $\frac{1}{2}+\frac{1}{3} = \frac{3}{6}+\frac{2}{6} = \boxed{\frac{5}{6}}$

5) $\frac{5}{4}\left(6-\frac{1}{5}\right) = \frac{30}{4} - \frac{5}{4}\frac{1}{5}$
$= \frac{30}{4} - \frac{1}{4}$
$= \boxed{\frac{29}{4}}$

B

1) $g(0) = (0)^2 - 3(0) + 1 = \boxed{1}$

2) $6g(2) = 6[(2)^2 - 3(2) + 1]$
$= 6[4-6+1]$
$= 6[-1] = \boxed{-6}$

3) $g(t) = \boxed{t^2 - 3t + 1}$

4) $g(t+2) = (t+2)^2 - 3(t+2) + 1$
$= t^2+4t+4-3t-6+1$
$= \boxed{t^2+t-1}$

5) $g(x+h) = (x+h)^2 - 3(x+h) + 1$
$= \boxed{x^2+2xh+h^2-3x-3h+1}$

9.3 Instantaneous Rates Intro (continued)

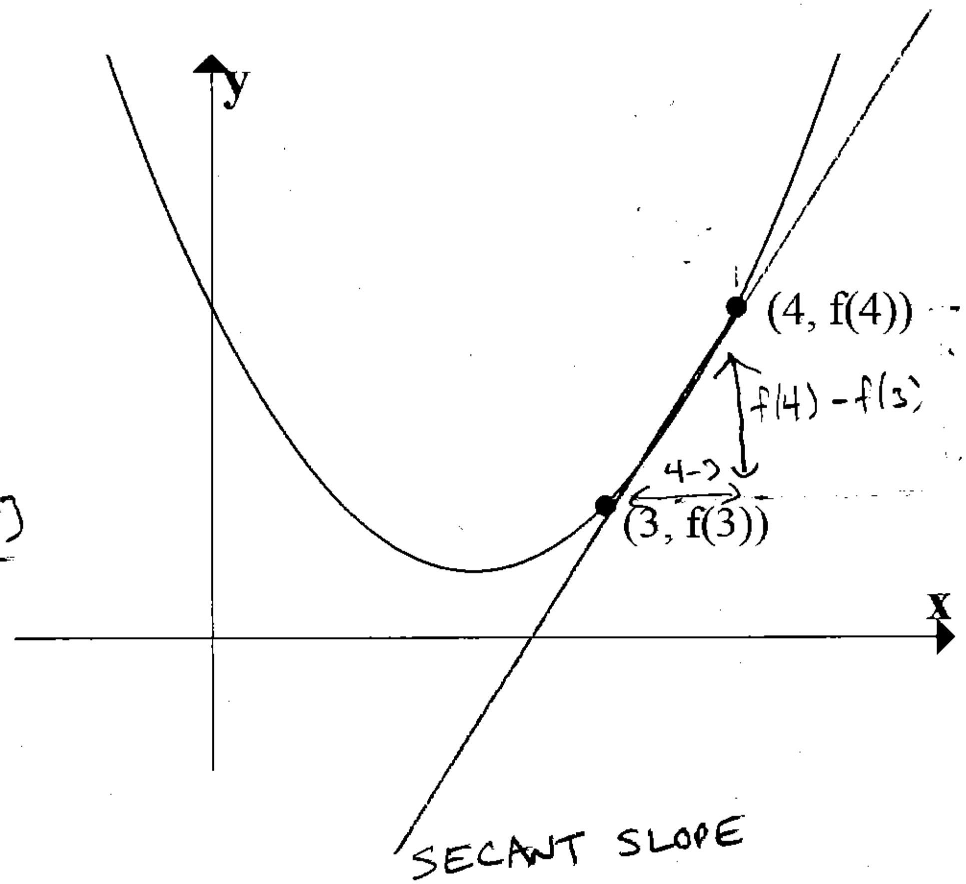
Let $f(x) = x^2 - 4x + 5$

You do: Find the average rate of change of $f(x)$ from $x = 3$ to $x = 4$.

$$\frac{f(4) - f(3)}{4 - 3} = \frac{[(4)^2 - 4(4) + 5] - [(3)^2 - 4(3) + 5]}{1}$$

$$= \frac{[16 - 16 + 5] - [9 - 12 + 5]}{1}$$

$$= \frac{5 - 2}{1} = \boxed{3}$$



Recall:

$f'(3)$ = *instantaneous rate at 3*

= *the slope of tangent at 3.*

Let's try to compute $f'(3)$.

Idea: Use a second point nearby

slope from 3 to 3.1	$\frac{f(3+0.1)-f(3)}{3.1-3}=\frac{[(3.1)^2-4(3.1)+5]-[(3)^2-4(3)+5]}{3.1-3}=\frac{2.21-2}{0.1}=\frac{0.21}{0.1}=2.1$
slope from 3 to 3.01	$\frac{f(3+0.01)-f(3)}{3.01-3}=\frac{[(3.01)^2-4(3.01)+5]-[(3)^2-4(3)+5]}{3.01-3}=\frac{2.0201-2}{0.01}=\frac{0.0201}{0.01}=2.01$
slope from 3 to 3.001	$\frac{f(3+0.001)-f(3)}{3.001-3}=\cdots=2.001$
slope from 3 to 3.0001	$\frac{f(3+0.0001)-f(3)}{3.0001-3}=\cdots=2.0001$

It appears the secant slope is getting closer and closer to 2!

Let's do this systematically:

A key shortcut: Instead of adding 0.1 or 0.01 or 0.001, let's label this amount by a symbol:

h = small number being added

Difference Quotient:For each approx., we compute

$$f'(3) \approx \frac{f(3+h) - f(3)}{(3+h) - 3} = ??$$

It becomes very easy to see the final answer if we can expand and simplify with algebra ***before*** plugging in numbers. Let's try it.

Recall: $f(x) = x^2 - 4x + 5$ What is $f'(3)$?

$$\boxed{f'(3) = 2}$$

Expand and completely simplify

$$\frac{f(3+h) - f(3)}{(3+h) - 3}$$

$$= \frac{[(3+h)^2 - 4(3+h) + 5] - [\underbrace{(3)^2 - 4(3) + 5}_{9 - 12 + 5 = 2}]}{(3+h) - 3}$$

$$= \frac{(3+h)^2 - 12 - 4h + 5 - 2}{h}$$

$$(3+h)^2 = (3+h)(3+h)$$
$$= 9 + 3h + 3h + h^2$$
$$= 9 + 6h + h^2$$

$$= \frac{\cancel{9} + 6h + h^2 - \cancel{12} - 4h + \cancel{3}}{h}$$

$$= \frac{6h + h^2 - 4h}{h}$$

$$= \frac{2h + h^2}{h} = \frac{h(2+h)}{h} = 2 + h$$

$$= \frac{2h}{h} + \frac{h^2}{h}$$

$$= 2 + h$$

THUS, $\boxed{\frac{f(3+h) - f(3)}{h} = 2 + h}$

COMPARE WITH TABLE!

AS $h \to 0$, THIS SLOPE $\to 2$

Same function: $f(x) = x^2 - 4x + 5$ What is $f'(5)$?

Find $f'(5)$ by using the same process.

Expand and completely simplify

$$\frac{f(5+h) - f(5)}{(5+h) - 5}$$

$$= \frac{[(5+h)^2 - 4(5+h) + 5] - \overbrace{[(5)^2 - 4(5) + 5]}^{25 - 20 + 5 = 10}}{(5+h) - 5}$$

$$= \frac{\cancel{25} + 10h + h^2 - \cancel{20} - 4h + \cancel{5} - \cancel{10}}{h}$$

$$= \frac{6h + h^2}{h}$$

$$= 6 + h$$

So $\boxed{f'(5) = 6}$

t is better just to do this once!

What is $f'(a)$?

Same function: $f(x) = x^2 - 4x + 5$

Find $f'(a)$ by using the same process.

Check: Does this match f'(3) and f'(5)?

$$\frac{f(a+h) - f(a)}{(a+h) - a}$$

$$= \frac{[(a+h)^2 - 4(a+h) + 5] - [a^2 - 4a + 5]}{a + h \quad - a}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 \cancel{-4a} - 4h \cancel{+5} \cancel{-a^2} \cancel{+4a} \cancel{-5}}{h}$$

$$= \frac{2ah + h^2 - 4h}{h}$$

$$= \frac{2ah}{h} + \frac{h^2}{h} - \frac{4h}{h}$$

$$= 2a + h - 4$$

So $\boxed{f'(a) = 2a - 4}$

$f'(3) = 2(3) - 4 = 2$ ✓

$f'(5) = 2(5) - 4 = 6$ ✓

YES!

Summary:

To get the formula for *the slope of the tangent line to f(x) at a point x.*

1. Completely simplify
$$\frac{f(x+h)-f(x)}{(x+h)-x}$$
2. Let *h* go to 0.
3. The resulting function is called the derivative function:
$$f'(x)$$
And it represents the *instantaneous rate of change of f(x) at x.*

We will learn a lot of "shortcuts" over the next week, but the process just described is the official definition of the derivative and we always come back to these ideas when we are thinking about new functions or when we are applying derivative concepts.